nitial and final pH might give greater decontamination. Accordingly, one additional level was assigned to each of the pH factors, namely, $A_4 = 10$ and $I_4 = 6$. The iron concentration factor was eliminated by holding it constant at the higher level.

A factorial experiment, devoted to the two pH factors, each at four levels, required $4 \times 4 = 16$ treatments, seven of which are new. All sixteen results are tabulated in Table 8. The expected improvement, from increasing the final pH to 10, is confirmed for all three initial pH levels. The advantage of increasing the initial pH to 6 is slight and appears only at the highest final pH.

CONCLUSIONS

The decontamination obtained with an initial pH of 6 and a final pH of 10, when an iron concentration of 1 mg./ml. and 0.5 g. of sulfide were used, was the best result observed in the study. Judging from results at the lower pH levels, halving of the amount of sulfide used has no influence on the results. The possibility of further improving the process with still higher values of the pH did not appear attractive because of the smallness of any anticipated effect and the increasing cost entailed by additional chemical treatment.

The value of using a sequence of progressively expanding and contracting factorial designs in a study devoted to the establishment of optimum operating conditions has been clearly demonstrated. Such a procedure generates a series of coherent conclusions characterized by maximum reliability and generality. While the goal of optimum conditions is attained in a highly efficient manner, the functional dependence of the dependent variable on those conditions is thoroughly explored.

ACKNOWLEDGMENT

The services of the following chemists are gratefully acknowledged: T. C. Tesdahl, T. G. Linxweiler, and D. R. Spangler.

Mound Laboratory is operated by Monsanto Chemical Company for the U. S. Atomic Energy Commission under contract AT-33-1-GEN-53.

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Presented at Nuclear Science and Engineering Congress, Cleveland.

A System for Counting Variables in Separation Processes

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In the proposed system for counting variables in separation processes the processes are resolved into their simpler component classes, e.g., theoretical plates, heat exchangers, reboilers, distillation columns, etc., and a distinction is made between those variables which are inherent in the systems and those which may be specified for design. Results are presented for the most commonly occurring component classes, and all possible process relations existing among these classes are expressed by a set of generalized equations [(16) to (19)]. The procedure of counting variables is therefore reduced to composition from variables for the component classes by use of the generalized equations.

In the design of processes for physical separation of components by mechanisms involving mass and heat transfer, the first step usually consists of specification of process conditions or independent variables. When the sufficient and necessary independent variables are fixed, the system is determined and other variables may be found by design computations. Normally the variables of a system are interrelated in such a way that only a few of them could be expressed as explicit functions of the others; the remaining ones have to be determined by lengthy calculations.

An example is the design of a distillation column separating a binary mixture of benzene and toluene. The column is to be designed to have one intermediate feed, a partial reboiler with a liquid-bottoms-product stream, and a total condenser with a liquid-distillate-product stream, and it will operate at atmospheric pressure. It is possible to specify for this column the concentration of either benzene or toluene in either the distillate or the bottoms stream, the recovery of either component in either stream, and the reflux ratio, viz., three independent variables. Then the number of theoretical plates both above and below the feed could be found by the familiar McCabe-Thiele diagram, thus determining two implicit dependent variables. A formal analysis shows, however, that for the column there exist four independent variables that could be specified. The fourth variable, not stated above, is implicit in the McCabe-Thiele method, that is, optimum location of the feed plate; the stepwise procedure is to be transferred from one operating line to the other in the vicinity of the intersection of the lines in order to secure a minimum total number of plates.

Another example is a so-called "double-distillation column" separating the ternary mixture, air—N_2, A, and O_2—with vapor air feed to the high-pressure column and a nitrogen-rich vapor distillate stream and an oxygen-rich vapor bottoms stream from the low-pressure column. In this case the number of independent variables that could be specified is not at all apparent. However, without a knowledge of the exact number of independent variables it is difficult, if not impossible, to proceed with a design problem in a systematic way. Often experience helps in setting trial values of certain variables very close to the correct answer, without the need of actually differentiating the implicit from the independent variables. With recent increase in use of electronic computers for design studies, however, it is desirable to know at the outset of a problem the correct number of independent variables as process conditions and to feed into the machine neither more nor less than those variables that can be specified, thus letting the machine perform the trial-and-error loops in finding the correct values of the dependent variables. In such a situation experience could hardly substitute for correct logic.

PRINCIPLES OF THE METHOD OF ANALYSIS

The difficulty of finding the correct number of independent variables was recognized by Gilliland and Reed (*2*), who proposed a method of attack by use of the phase rule and the first law of thermodynamics. Other discussions on the subject can be found in the literature

Tabular material has been deposited as document 4807 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for \$5.00 for photoprints or \$2.25 for 35-mm. microfilm.

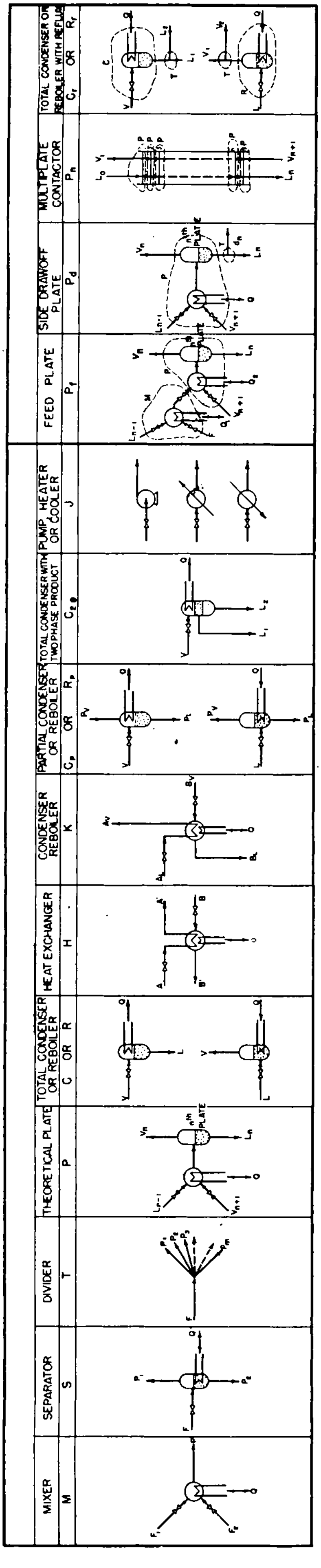

Fig. 1. Elements.

Fig. 2. Complex elements.

(*1, 3*). As will be shown below, the phase rule contributes toward accounting for all variables, N_v, in a system, and the first law toward accounting for all possible conditions, N_c, inherent and necessary in the system. The difference between the possible variables and the possible conditions represents the independent variables, N_i. Thus

$$N_i = N_v - N_c \qquad (1)$$

The possible variables of a system, N_v, could be enumerated as follows:

1. The phase rule gives the degree of freedom of any single-stage system in which there exists either one phase or two or more phases in equilibrium:

$$N = C + 2 - \phi \qquad (2)$$

These degrees of freedom represent the so-called "intensive" variables such as concentration, temperature, pressure, entropy, and other thermodynamic properties determined by the state and independent of the quantities of the components present. For a flow system, however, there is associated with any stream an additional extensive variable —its rate of flow—which is not determined by the state and is therefore not dealt with by the phase rule itself. Therefore, for a one-phase system there are $C + 1$ intensive variables, 1 extensive variable, or $C + 2$ total variables. For a two-phase system, where the flow rates of both phases could be set independently, there are C intensive variables, 2 extensive variables, and $C + 2$ total variables.

2. For any system, considered as a whole, or any part thereof, there remains, in addition to the foregoing variables, the degree of freedom of choosing the amount, or rate, of energy exchanged between the system and its surroundings.

In a strict sense the method is applicable to systems in equilibrium, inasmuch as the basis of the phase rule is equilibrium. Therefore the analysis will be rigorously correct for separation processes reducible to a stagewise nature. Differential types of operation should be viewed as consisting of an infinite number of stages each of which approaches some presupposed percentage of equilibrium.

The conditions inherent and necessary in a system, N_c, are enumerated as follows:

1. A system has to be in material and energy balance. The first law of thermodynamics states that the total amount of energy entering any system must be exactly equal to that leaving plus any accumulation of energy within the system. For flow processes in which changes in kinetic energy, potential energy, and work done are negligible, the first law is simplified to straightforward enthalpy, or heat, balance. Normally heat balance determines one condition in any system, and as many conditions are fixed by material balances as there are components in the system.

2. Additional conditions inherent in a system are found in equality of variables of certain streams, such as those joining different parts of a composite system.

For ordinary operations certain of the variables are often set by design, such as the composition and flow rate of the feeds, the pressure on each plate of a distillation column, and the heat leak to or from each plate. Thus for any system the number of these normally fixed variables is

$$N_x = N_P + N_\pi + N_Q \qquad (3)$$

The difference between the independent variables, N_i, and those normally fixed, N_x, stands for the independent variables available, N_a, for process specifications:

$$N_a = N_i - N_x \qquad (4)$$

$$= N_v - N_c - N_x \qquad (5)$$

The available independent variables, N_a, are the principal ones of interest in the solution of design problems, and the present method is devoted mainly to their enumeration.

A SYSTEMATIC APPROACH

The basic principles set forth above are relatively simple, and in theory the desired number of variables could always be obtained by starting from these fundamentals. In practice, it will be found that such a procedure is not only tedious but also sometimes confusing. One frequent difficulty in applying the foregoing principles lies in the recognition of variables and conditions so that none is overlooked or counted twice. As an example, from the use of the McCabe-Thiele method of designing distillation columns one is likely to reach the erroneous conclusion that only three independent variables exist in a distillation column with an intermediate feed. As indicated, the correct number is four. As a further example the heat input to and heat leak from a reboiler may be considered. The question arises of whether they are two independent variables or just one. Sometimes it is not easy to see whether certain conditions should be classed as inherent in a system or as normally fixed in design. For instance, when a stream is divided the operation is adiabatic. The fact of no heat exchange between the system and its surrounding would appear either as an inherent condition of the system or as a normally fixed variable representing zero heat leak.

The present paper presents a self-consistent system of classifying and accounting variables and conditions, designed to avoid the errors and dilemmas described above. The results of this study are tabulated and summarized so that any complex system may be analyzed with a minimum of time without resorting to first principles.

PROPOSED METHOD

In the present method the component parts of a separation process or a system of operations are classified as elements, complex elements, units, and complex units, in the order of increasing complex-

TABLE 1. ELEMENTS

Element Symbol / Variables	Description	Number	Mixer M	Separator S	Divider T	Theoretical plate P	Total condenser or reboiler C or R	Heat exchanger H	Condenser reboiler K	Partial condenser or reboiler(7) C_p or R_p	Total condenser with two-phase product $C_{2\phi}$	Pump, heater or cooler J
No. of streams	$1-\phi$	—	3	1	$m+1$	2	2	4	4	1	1	2
	$2-\phi$	—	0	1	0	1	0	0	0	1	1	0
N_v^e	$1-\phi$	$C+2$	$3(C+2)$	$C+2$	$(m+1)(C+2)$	$2(C+2)$	$2(C+2)$	$4(C+2)$	$4(C+2)$	$C+2$	$C+2$	$2(C+2)$
	$2-\phi$	$C+2$	0	$C+2$	0	$C+2$	0	0	0	$C+2$	$C+2$	0
	Q	1	1	1	1	1	1	1	1	1	1	1
	Total		$3C+7$	$2C+5$	$mC+C+2m+3$	$3C+7$	$2C+5$	$4C+9$	$4C+9$	$2C+5$	$2C+5$	$2C+5$
N_c^e	Material balance	C	C	C	C	C	C	$2C$(4)	$2C$(5)	C	C	C
	Heat balance	1	1	1	1	1	1	1	1	1	1	1
	Q	1	—	—	$1(Q=0)$	—	$1(Q=$ latent heat$)$(3)	—	—	—	$1(Q=$ latent heat$)$(3)	—
	Equality	—	—	—	(1)$(m-1)(C+1)+1$(2)	—	—	—	2(6)	—	—	—
	Total		$C+1$	$C+1$	$mC+m+2$	$C+1$	$C+2$	$2C+1$	$2C+3$	$C+1$	$C+2$	$C+1$
$N_i^e = N_v^e - N_c^e$		—	$2C+6$	$C+4$	$C+m+1$	$2C+6$	$C+3$	$2C+8$	$2C+6$	$C+4$	$C+3$	$C+4$
N_x^e	Feed	$C+2$	$2(C+2)$	$C+2$	$C+2$	$2(C+2)$	$C+2$	$2(C+2)$	$2(C+2)$	$C+2$	$C+2$	$C+2$
	π	1	1	1	—	1	1	2	2	1	1	1(8)
	Q	1	1	1	—	1	—	1	1	—	—	
	Total		$2C+6$	$C+4$	$C+2$	$2C+6$	$C+3$	$2C+7$	$2C+7$	$C+3$	$C+3$	$C+2$
$N_a^e = N_i^e - N_x^e$			0	0	$m-1$	0	0	1	-1	1	0	1
Example of N_a^e								Temp. of A' or B'	For assigned heat leak and pressures of streams A and B both flow rates of A and B cannot be fixed at same time.	Q or P_L/P_v ratio		Discharge pressure or temperature

NOTES:
(1) Equality of intensive variables of all product streams.
(2) Pressure of system equals pressure of feed.
(3) Net heat exchange with surrounding including latent heat and heat leak.
(4) Since there is no material exchange between streams AA' and BB', there are C material balances for each stream, or altogether $2C$ material balances.
(5) Since there is no material exchange between streams A_LA_v and B_vB_L, there are C material balances for each stream, or altogether $2C$ material balances.
(6) Heat exchanged equal to latent heat of vaporization of both streams.
(7) An evaporator is a special case of R_p.
(8) Constant heat leak for pump, or constant pressure for heater and cooler.

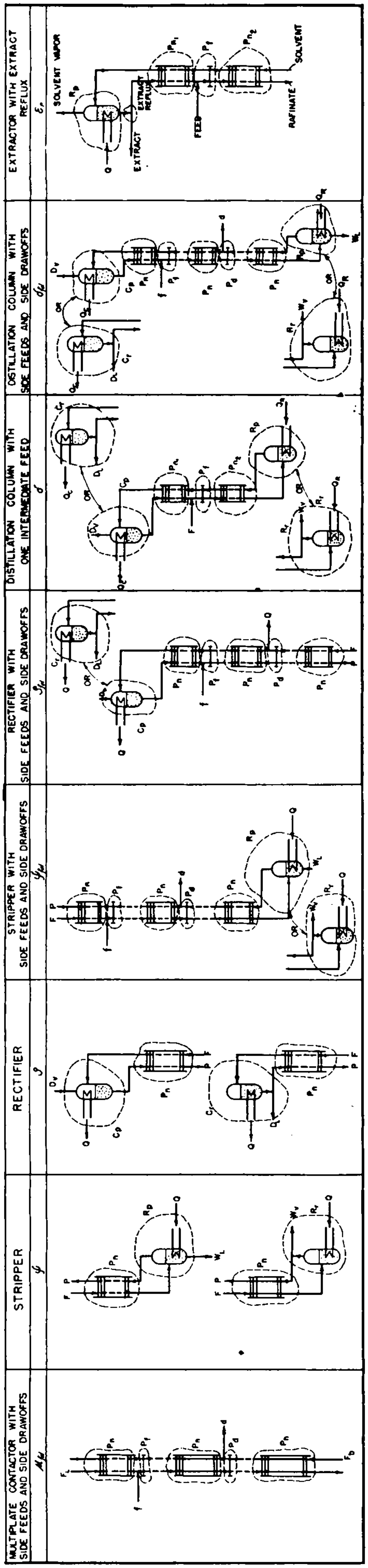

Fig. 3. Units.

ity. The classification is somewhat arbitrary and is made for convenience' sake. An element is a single-stage equipment, such as a flash drum, a total condenser, a theoretical plate, etc. A complex element is an element with certain minor additional features, such as a total condenser with reflux—that is, a total condenser from which the condensate is divided into a distillate stream and a reflux stream. Another example of a complex element is a repetition of theoretical plates in series so as to form a multiplate contactor, a particular case of which is found in an absorber. A combination of elements or complex elements gives rise to a unit, such as a stripping column, which is a multiplate contactor combined with a reboiler. When units, complex elements, or elements are connected, they form complex units, such as a petroleum fractionator with strippers attached to side drawoff streams. It can be seen that a more complex, or major, class (such as a stripping column) is composed of less complex, or minor, classes (theoretical plates and reboiler of the stripping column).

The problem of enumerating variables and conditions is therefore reduced to the following two stages:

1. Enumeration of variables and conditions of the simplest class, the element, by direct application of the principles adopted.
2. Establishment of relations among the various classes so that the major classes might be analyzed by applying the results obtained for the minor classes without resort to the first principles.

Throughout this study the normally fixed variables for any system will be limited to the composition and flow rate of the feed,* the pressure of the system, and heat exchanged between the system and its surroundings. This selection of fixed variables is arbitrary, but the logic of the method is hardly affected by any other selection.

Elements

For the first stage of the problem, a start will be made with the element. The number, N, representing variables or conditions, will be assigned a superscript to designate one of the four classes listed above and a subscript denoting the nature of the variable or condition. For example, N_i^e denotes the number of *independent* variables for an *element*. Equations (1), (4), and (5) could be written for any element as follows:

$$N_i^e = N_v^e - N_c^e \qquad (6)$$

$$N_a^e = N_i^e - N_x^e \qquad (7)$$

$$= N_v^e - N_c^e - N_x^e \qquad (8)$$

*Exceptions will be found in which the flow rate of a feed is not normally fixed. For instance, the flow rate of the strippant in a stripper is usually considered a variable available for process specification. In this case one more variable will therefore have to be added to the number resulting from the present analysis.

An example of an element is the case of a theoretical plate, P. Two single-phase streams containing C components each are fed to the plate in the form of liquid from the plate above and vapor from the plate below. The total number of variables of these two streams is $2(C + 2)$. Leaving the plate are two streams in equilibrium, vapor to the plate above and liquid to the plate below. These two streams in equilibrium are counted as a two-phase system representing $C + 2$ variables. Another variable is found in any heat exchange with the surroundings. Therefore the total number of variables is

$$N_v^e = 2(C + 2) + (C + 2) + 1$$
$$= 3C + 7$$

The inherent conditions are material balances for the C components and one heat balance of all the streams entering and leaving the plate:

$$N_c^e = C + 1$$

Therefore according to Equation (6)

$$N_i^e = (3C + 7) - (C + 1)$$
$$= 2C + 6$$

The usually fixed variables are the compositions and flow rates of the two feeds, $2(C + 2)$, and the pressure on plate, 1, and the heat exchange (heat leak) with the surrounding, 1, or

$$N_x^e = 2(C + 2) + 1 + 1$$
$$= 2C + 6$$

Therefore

$$N_a^e = (2C + 6) - (2C + 6)$$
$$= 0$$

A little reflection will show that a constant-pressure theoretical plate with fixed vapor and liquid feeds and fixed heat leak is a nonvariant system.

Table 1† repeats the foregoing analysis for a number of representative elements, viz., mixer, separator, divider, condenser or reboiler, total condenser with two-phase product, and pump, heater, or cooler, all of which are illustrated schematically in Figure 1. Each element is assigned a symbol, such as P for the theoretical plate, and is described in a sketch in Figure 1. In what follows it will be seen that the results set forth in Table 1 may be used directly in deriving variables of more complex classes without need to refer to first principles.

Complex Elements

A complex element is essentially a modified element, the modification usually being the addition or division of a

†See footnote on page 240.

TABLE 2. COMPLEX ELEMENTS

Complex element Symbol		Feed plate P_f	Side drawoff plate P_d	Multiplate contactor P_n	Total condenser or reboiler with reflux C_r or R_r
Variables: Description	Number				
No. of streams: Feed		3	2	2	1
No. of streams: inter-		1	1	$2(n-1)$	1
No. of elements: M		1	0	0	0
No. of elements: T		0	1	0	1
No. of elements: P		1	1	n	0
No. of elements: C or R		0	0	0	1
No. of plates		1	1	n	—
N_v^E: M	$2C+6$	$2C+6$	—	—	—
N_v^E: T	$C+m+1$	—	$C+3$	—	$C+3$
N_v^E: P	$2C+6$	$2C+6$	$2C+6$	$n(2C+6)$	—
N_v^E: C or R	$C+3$	—	—	—	$C+3$
N_v^E: α	1	0	0	1	—
N_v^E: Total		$4C+12$	$3C+9$	$2nC+6n+1$	$2C+6$
N_c^E = Interstreams	$C+2$	$C+2$	$C+2$	$2(n-1)(C+2)$	$C+2$
$N_i^E = N_v^E - N_c^E$		$3C+10$	$2C+7$	$2n+2C+5$	$C+4$
N_x^E: N_F^E	$C+2$	$3(C+2)$	$2(C+2)$	$2(C+2)$	$C+2$
N_x^E: $N_x^e - N_F^e$: M	2	2	—	—	—
N_x^E: $N_x^e - N_F^e$: T	0	—	0	—	0
N_x^E: $N_x^e - N_F^e$: P	2	2	2	$2n$	—
N_x^E: $N_x^e - N_F^e$: C or R	1	—	—	—	1
N_x^E: Total		$3C+10$	$2C+6$	$2n+2C+4$	$C+3$
$N_a^E = N_i^E - N_x^E$		0	1	1	1
Example of N_a^E			Flow rate of side drawoff	Number of plates or concentration of any component in either of two product streams.	Reflux ratio or Q

stream. For a complex element, Equations (1), (4), and (5) can be written as follows:

$$N_i^E = N_v^E - N_c^E \tag{9}$$

$$N_a^E = N_i^E - N_x^E \tag{10}$$

$$= N_v^E - N_c^E - N_x^E \tag{11}$$

Since a complex element is made up of a combination of elements, the following relation between the complex element and its component elements holds:

$$N_v^E = \sum N_i^e + N_\alpha \tag{12}$$

This equation states that the *total* number of variables of a complex element is equal to the sum of the *independent* variables of the component elements, plus N_α, which stands for the freedom of choice of the number of times which any component element could be repeated. The variable, N_α, is not equal to the number of such repetitions but represents the single degree of freedom with which the number of such repetitions could be chosen. By means of Equation (12), the inherent conditions in the component elements which are already covered in $\sum N_i^e$ will not be counted again as conditions specific to the complex element. This illustrates a rule of counting variables devised to avoid redundancy. It will be referred to as the rule of coverage, which will be used repeatedly in this study.

The conditions inherent and necessary in the complex element are limited to those streams interconnecting the component elements. These streams will be referred to as *interstreams*. When two elements are connected by an interstream, the stream leaving one element and the stream entering the other, which are considered as two independent streams when the elements are treated separately, necessarily possess the same intensive and extensive variables. Such equality in the intensive and extensive variables constitutes the inherent and necessary conditions of an interstream. If an interstream is single phase, these conditions stand for $C + 2$ variables.

In counting the normally fixed variables for a complex element, the following relation is adopted:

$$N_x^E = N_F^E + \sum (N_x^e - N_F^e) \tag{13}$$

This equation divides the normally fixed variables into two groups, those pertaining to the feed, N_F, and those belonging to pressure and heat leak, $N_x - N_F$. Since a complex element and its component element will often have common feeds, the foregoing division is designed to avoid redundancy in enumeration. This measure is arbitrary, but as long as it is followed consistently the present system of counting variables will maintain its validity.

Substitution of Equations (12) and (13) in Equations (9) and (11) yields

$$N_i^E = \sum N_i^e + N_\alpha - N_c^E \tag{14}$$

$$N_a^E = \sum N_i^e + N_\alpha - N_c^E - N_F^E - \sum (N_x^e - N_F^e) \tag{15}$$

As an example of a complex element one may consider a multiplate contactor, of which a gas absorber is a particular case. (*See* Table 2*.) With the number of plates equal to n, from Table 1, for a theoretical plate, $N_i^e = 2C + 6$. For n theoretical plates the number of variables is $n(2C + 6)$. Since the number of plates, n, could be specified by design, the specification of n represents one additional independent variable, or $N_\alpha = 1$.

*See footnote on page 240.

Table 3. Summary

Class	*System*	*Symbol*	N_i	$N_x - N_F$	N_x	N_a
Elements	Mixer	M	$2C+6$	2	$2C+6$	0
	Separator	S	$C+4$	2	$C+4$	0
	Divider	T	$C+m+1$	0	$C+2$	$m-1$
	Theoretical plate	P	$2C+6$	2	$2C+6$	0
	Total condenser or reboiler	C or R	$C+3$	1	$C+3$	0
	Heat exchanger	H	$2C+8$	3	$2C+7$	1
	Condenser reboiler	K	$2C+6$	3	$2C+7$	-1
	Partial condenser or reboiler	C_p or R_p	$C+4$	1	$C+3$	1
	Total condenser with two-phase product	$C_{2\phi}$	$C+3$	1	$C+3$	0
	Pump, heater, or cooler	J	$C+4$	1	$C+3$	1
Complex elements	Feed plate	P_f	$3C+10$	4	$3C+10$	0
	Side drawoff plate	P_d	$2C+7$	2	$2C+6$	1
	Multiplate contactor	P_n	$2n+2C+5$	$2n$	$2n+2C+4$	1
	Total condenser or reboiler with reflux	C_r or R_r	$C+4$	1	$C+3$	1
	Multiplate contactor with side feeds and side drawoffs	μ_{fd}	$2\sum_{1}^{f+d+1} n_i+f(C+7)+4d+2C+5$	$2\sum_{1}^{f+d+1} n_i+4f+2d$	$2\sum_{1}^{f+d+1} n_i+f(C+6)+2d+2C+4$	$f+2d+1$
	Stripper	ψ	$2n+C+5$	$2n+1$	$2n+C+3$	2
	Rectifier	ρ	$2n+C+5$	$2n+1$	$2n+C+3$	2
	Stripper with side feeds and side drawoffs	ψ_{fd}	$2\sum_{1}^{f+d+1} n_i+f(C+7)+4d+C+5$	$2\sum_{1}^{f+d+1} n_i+4f+2d+1$	$2\sum_{1}^{f+d+1} n_i+f(C+6)+2d+C+3$	$f+2d+2$

Units	Rectifier with side feeds and side drawoffs	ρ_{fd}	$2\sum_{1}^{f+d+1} n_i+f(C+7)+4d+C+5$	$2\sum_{1}^{f+d+1} n_i+4f+2d+1$	$2\sum_{1}^{f+d+1} n_i+f(C+6)+2d+C+3$	$f+2d+2$
	Distillation column with one intermediate feed	δ	$2(n_1+n_2)+C+12$	$2(n_1+n_2)+6$	$2(n_1+n_2)+C+8$	4
	Distillation column with side feeds and side drawoffs	δ_{fd}	$2\sum_{1}^{f+d+1} n_i+f(C+7)+4d+5$	$2\sum_{1}^{f+d+1} n_i+4f+2d+2$	$2\sum_{1}^{f+d+1} n_i+f(C+6)+2d+2$	$f+2d+3$
	Extractor with extract reflux	ϵ_r	$2(n_1+n_2)+2C+13$	$2(n_1+n_2)+5$	$2(n_1+n_2)+2C+9$	4
Complex units	Distillation columns in series	$\delta+\delta$	$2\sum_{1}^{4} n_i+C+22$	$2\sum_{1}^{4} n_i+12$	$2\sum_{1}^{4} n_i+C+14$	8
	Distillation column with auxiliary stripper or rectifier	$\delta+(\psi)_{aux.}$ or $\delta+(\rho)_{aux.}$	$2\sum_{1}^{5} n_i+C+24$	$2\sum_{1}^{5} n_i+13$	$2\sum_{1}^{5} n_i+C+15$	9
	Distillation column with auxiliary complete column	$\delta+(\delta)_{aux.}$	$2\sum_{1}^{6} n_i+C+31$	$2\sum_{1}^{6} n_i+18$	$2\sum_{1}^{6} n_i+C+20$	11
	Simple double column	$(\delta)^2$	$2(n_1+n_2+n_3)+C+13$	$2(n_1+n_2+n_3)+7$	$2(n_1+n_2+n_3)+C+9$	4
	Double column with side feeds and side drawoffs	$(\delta_{fd})^2$	$2\sum_{1}^{\sigma+2} n_i+(f_l+f_h+1)(C+7)+4(d_l+d_h)+6$	$2\sum_{1}^{\sigma+2} n_i+4(f_l+f_h+1)+2(d_l+d_h)+3$	$2\sum_{1}^{\sigma+2} n_i+(f_l+f_h+1)(C+6)+2(d_l+d_h)+3$	$f_l+f_h+2(d_l+d_h)+4$
	Binary azeotropic distillation column	$(\delta)_{2\phi}$	$2(n_1+n_2)+C+11$	$2(n_1+n_2)+6$	$2(n_1+n_2)+C+8$	3
	Twin azeotropic distillation columns	$(\delta+\psi)_{2\phi}$	$2(n_1+n_2+n_3)+C+16$	$2(n_1+n_2+n_3)+9$	$2(n_1+n_2+n_3)+C+11$	5
	Absorption–desorption cycle (2 pressures; 1 temp.)	—	$2(n_1+n_2)+2C+8$	$2(n_1+n_2+1)$	$2(n_1+n_2)+2C+5$	3*
	Absorption–desorption cycle (2 pressures; 2 temp.)	—	$2(n_1+n_2)+2C+16$	$2(n_1+n_2)+6$	$2(n_1+n_2)+2C+10$	6*

*See footnote * on page 243.

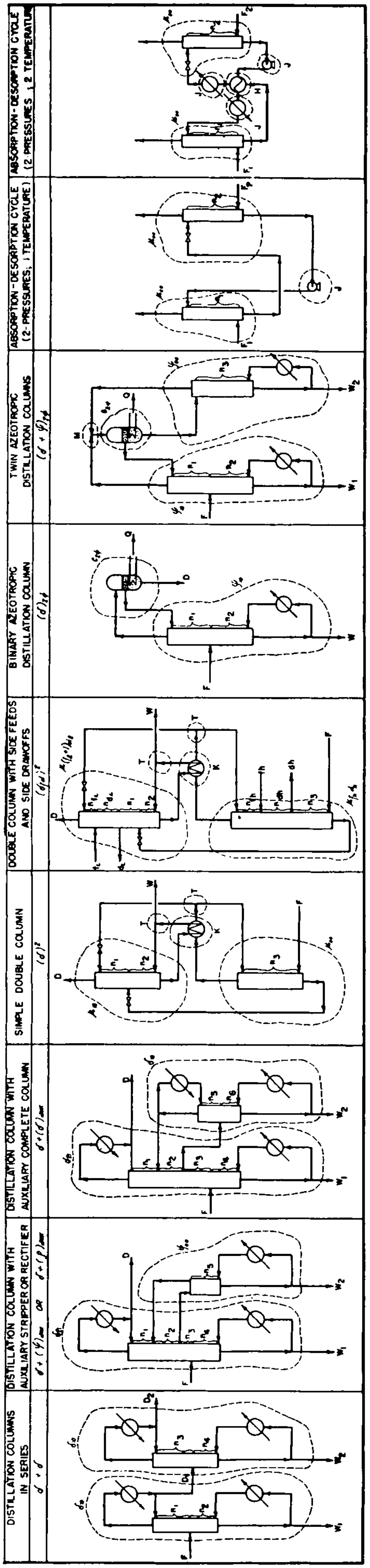

Fig. 4. Complex units.

Therefore

$$N_v^E = n(2C + 6) + 1$$

Every two adjacent plates are connected by two interstreams, each representing $C + 2$ conditions, or $N_c^E = C + 2$ per interstream. As there are $2(n - 1)$ interstreams for the n theoretical plates, there are $2(n - 1)(C + 2)$ inherent conditions associated with these interstreams, or

$$N_c^E = 2(n - 1)(C + 2)$$

$$N_i^E = n(2C + 6) + 1 - 2(n - 1)(C + 2) = 2C + 2n + 5$$

The conditions normally fixed for each theoretical plate are its pressure, 1, and heat leak, 1 (*see* Table 1 and Figure 2) or 2 per plate. Therefore for n plates

$$\sum (N_x^e - N_F^e) = 2n$$

The variables set by specifying the two feed streams to the multiplate contactor are

$$N_F^E = 2(C + 2)$$

Therefore

$$N_x^E = 2(C + 2) + 2n$$

and

$$N_a^E = [2C + 2n + 5] - [2(C + 2) + 2n] = 1$$

One way of interpreting this single available independent variable, $N_a^E = 1$, is that for two given feeds the two product streams are entirely determined if the number of plates of an absorber is specified.

Table 2 repeats the foregoing analysis for representative complex elements, viz., feed plate, side-drawoff plate, multiplate contactor, and total condenser or reboiler with reflux. These complex elements are shown in Figure 2.

Units and Complex Units

In a similar manner, it can be shown that the following relations hold between major and minor classes:

$$N_v^y = \sum_{minor} N_i + N_\alpha \qquad (16)$$

$$N_i^y = \sum_{minor} N_i + N_\alpha - N_c^y \qquad (17)$$

$$N_x^y = N_F^y - \sum_{minor} (N_x - N_F) \qquad (18)$$

$$N_a^y = \sum_{minor} N_i + N_\alpha - N_c^y - N_F^y - \sum_{minor} (N_x - N_F) \qquad (19)$$

where the superscript y, stands for any major class and $\sum_{minor}$ for the appropriate summation of all classes minor to y.

Figures 3 and 4 illustrate representative units and complex units respectively. The results of analysis of Figures 1 to 4 are summarized in Table 3* in a form convenient for carrying out the variable-counting procedure. Table 3 includes the more important components of separation processes frequently encountered in chemical plant design.

Examples

The application of the preceding general relations will be illustrated with two examples, one for a unit and one for a complex unit.

An example of a unit is a conventional distillation column, δ, consisting of an intermediate feed, a total condenser with reflux and liquid distillate product (or a partial condenser with vapor distillate product), and a total reboiler with vapor bottoms product (or a partial reboiler with liquid bottoms product). Such a column is shown as the sixth case, δ, in Figure 3.

Before the appropriate equations are applied to an analysis of the preceding case, it is desirable to define precisely the minor classes of which the system consists, the interstreams among these minor classes, and the streams entering and leaving the system as a whole. The present system could be broken into the following minor classes:

one feed plate, P_f
two multiplate contactors, P_n
one partial condenser, C_p, or one total condenser with reflux, C_r
one partial reboiler, R_p, or one total reboiler with reflux, R_r

Reference to the sketch of this system shown in Figure 3 indicates one feed and eight interstreams.

Therefore according to Equation (16) the total number of variables is as follows: (*See* results given in Table 1.)

one feed plate, P_f: $3C + 10$
two multiplate contactors, P_n: $2(n_1 + n_2) + 2(2C + 5)$
one condenser, C_p or C_r: $C + 4$
one reboiler, R_p or R_r: $C + 4$
N_α: 0 (since no component class is repeated)
Therefore $N_v^u = 2(n_1 + n_2) + 9C + 28$

The only conditions inherent in combining the foregoing minor classes are those offered by the eight interstreams, or

$$N_c^u = 8(C + 2)$$

Therefore the total number of independent variables is, according to Equation (17),

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*See footnote on page 240.

$$N_i^u = [2(n_1 + n_2) + 9C + 28]$$
$$- 8(C + 2) = 2(n_1 + n_2) + C + 12$$

The normally fixed variables of the minor classes, except feed, are as follows: (*See* Table 2.)

one feed plate, P_f: 4
two multiplate contactors, P_n: $2(n_1 + n_2)$
one condenser, C_p or C_r: 1
one reboiler, R_p or R_r: 1

$$\sum_{minor} (N_x - N_F) = 2(n_1 + n_2) + 6$$

The normally fixed variables for the unit are, according to Equation (18), greater than the preceding by those variables represented by the feed into the unit, or $C + 2$. Therefore

$$N_x^u = (C + 2) + [2(n_1 + n_2) + 6]$$
$$= 2(n_1 + n_2) + C + 8$$

And, according to Equation (19),

$$N_a^u = [2(n_1 + n_2) + C + 12]$$
$$- [2(n_1 + n_2) + C + 8]$$
$$= 4$$

This number of 4 was discussed in the introduction, and the independent variables often chosen to give this number will not be repeated here. One fact which is of interest is that the number of normally available independent variables, $N_a^u = 4$, is not affected by the number of components, C, which the unit involves. This fact would be found true for the even more complex combinations of the various classes, as may be seen in Table 3.

As a second example illustrating the application of the general relations given by Equations (16), (17), (18), and (19), one may consider a case of the complex unit—an absorption-desorption cycle operating at two pressures and two temperatures. Such an operation is exemplified in the purification of gases by removal of its carbon dioxide content with an ethanolamine solution. The carbon dioxide is absorbed by the ethanolamine solution in an absorber at a lower temperature and higher pressure. The dissolved carbon dioxide in the ethanolamine solution is removed in a stripper by steam at a higher temperature and lower pressure. Interconnecting the absorber and stripper are heat exchangers for cooling and warming the liquid streams and a circulating pump. This cycle is given as the last case of the complex units in Figure 4. The cycle could be broken into the following minor classes:

one cooler, one heater, and one pump: J
one heat exchanger: H
two multiplate contactors with no side feeds and side drawoffs: μ_{00}

The sketch in Figure 4 shows two feeds and seven interstreams. According to Equation (16) the total number of variables is

one cooler, one heater, and one pump: $3(C + 4)$
one heat exchanger: $2C + 8$
two simple multiplate contactors: $2(n_1 + n_2) + 4C + 10$
N_α: 0

$$N_v^U = 2(n_1 + n_2) + 9C + 30$$

The inherent conditions are represented by the seven interstreams:

$$N_c^U = 7(C + 2)$$

Therefore the total number of independent variables is, according to Equation (17),

$$N_i^U = 2(n_1 + n_2) + 9C + 30$$
$$- 7(C + 2)$$
$$= 2(n_1 + n_2) + 2C + 16$$

The normally fixed variables, except feed, of the minor classes are

one cooler, one heater, and one pump: 3
one heat exchanger: 3
two simple multiplate contactors: $2(n_1 + n_2)$

$$\sum_{minor} (N_x - N_F) = 2(n_1 + n_2) + 6$$

The variables fixed by fixing the two feed streams entering the absorption-desorption cycle are $2(C + 2)$.

Therefore

$$N_x^U = 2(C + 2) + 2(n_1 + n_2) + 6$$
$$= 2(n_1 + n_2) + 2C + 10$$

According to Equation (19)

$$N_a^U = [2(n_1 + n_2) + 2C + 16]$$
$$- [2(n_1 + n_2) + 2C + 10]$$
$$= 6$$

The following represents a possible combination of the six variables available for process specification:

1. Concentration of a key component in the stream leaving the absorber.
2. Concentration of this key component in the lean liquor entering the absorber.
3. Circulation rates of the absorbing liquor.
4. Rate of heat exchange in heat exchanger.
5. Temperature approach to cooling water in cooler.
6. Temperature approach to heating medium in heater.

As noted earlier*, an exception to normally fixing the flow rate of a feed is found in the flow rate of a strippant in a stripper. Therefore, there is a seventh variable available for process specification, viz., the strippant flow rate.

*See footnote on page 243.

ACKNOWLEDGMENT

The author gratefully acknowledges the criticism and changes made by A. M. Squires and Manson Benedict on the original manuscript.

NOTATION

C = condenser; also component
d = side drawoff
D = distillate stream
f = side feed
F = feed, terminal
H = heat exchanger
J = pump, heater or cooler
K = condenser reboiler
L = liquid stream
m = number of divided stream
M = mixer
n = number of theoretical plates
N = number of variables or conditions
p = product
P = theoretical plate
Q = heat exchanged between system and surroundings
R = reboiler
S = separator
T = divider
V = vapor stream
w = bottoms stream
δ = distillation column
ϵ = extraction column
μ = multiplate contactor
π = pressure
ρ = rectifier
σ = $f + d + 1$
ϕ = phase
ψ = stripper

Subscripts

a = available variables for process specification
c = conditions inherent in system
d = with side drawoffs
f = with side feeds
F = feeds
h = high-pressure column of δ^2
i = total independent variables
l = low-pressure column of δ^2
Q = heat exchanged between system and surroundings
v = total variables of system
x = normally fixed variables
α = repetitions of elements
π = pressure

Superscripts

e = element
E = complex element
u = unit
U = complex unit

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